5.3 Optimization

H Physics

Ex 1. The radial probability density (for $n=2$, $l=1$, $m=0$) for the hydrogen atom can be written as:

$$P(r)=Cr^4e^{-r/a}$$

where C is a constant, and $a=0.529\times10^{-10}m$.

Find the most likely position (r) of the electron (find the radius r where $P(r)$ is maximum).

$P'(r)=C(4)r^3e^{-r/a}+Cr^4e^{-r/a}(-1/a)$

$=Cr^3e^{-r/a}(4-r/a)$

$P'(r)=0 \quad when \quad 4=r/a$

$r=4a=4(0.529\times10^{-10}m)=2.116\times10^{-10}m$

$\therefore$ The most likely position is $2.116\times10^{-10}m$ from the center of the hydrogen atom.

Ex 2. The position function fro a particle is given by:

$s(t)=2te^{-t}, \quad t\geq0$.

a) Find the maximum value for the position s.

$s'(t)=2e^{-t}-2te^{-t}=2(1-t)e^{-t}$

$s'(t)=0 \; at \; t=1$

$s(1)=2e^{-1}=2/e$

t	0		1	
$s(t)$		↗	$2/e$	↘
$s'(t)$		+	0	-

$\therefore$ The maximum value of the position s is $2/e$.

b) Find the minimum value for the velocity $v=s'$.

$v=s'(t)=2(1-t)e^{-t}$

$v'=-2e^{-t}-2(1-t)e^{-t}=e^{-t}(-2-2+2t)=-2(2-t)e^{-t}$

$v'=0 \; at \; t=2$

$v(2)=-2e^{-2}$

t	0		2	
$v(t)$		↘	$-2/e^2$	↗
$v'(t)$		-	0	+

$\therefore$ The minimum value of velocity is $-2/e^2$.

Reading: Nelson Textbook, Pages 241-244 (Optimization Problems Involving Exponential Functions)
Homework: Nelson Textbook: Page 245 #3, 5, 6, 10, 11, 12, 13, 14