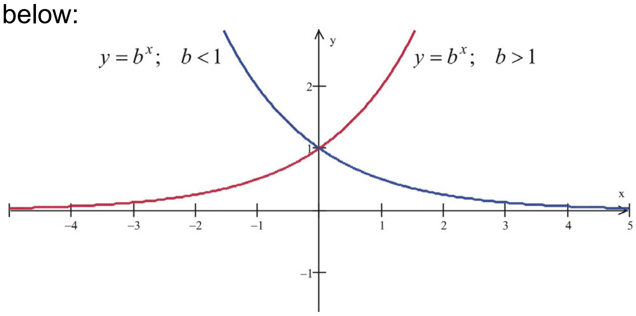
5.1 5.2 Derivative of Exponential Function

A Review of Exponential Functions

The exponential function is defined as:

$$y = f(x) = b^x; \quad b > 0, b \neq 1$$

The graph of the exponential function is represented below:



The x-axis ($y = 0$) is a horizontal asymptote.

Ex 1. Use the graph of the exponential function to evaluate each limit.

a) $\lim_{x\to\infty} 2^x$

b) $\lim_{x\to-\infty} 2^x$

c) $\lim_{x\to\infty} 0.2^x$

d) $\lim_{x\to-\infty} 0.2^x$

B Number e

The number e is defined by:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad (1)$$

which can be written also as:

$$e = \lim_{u\to 0}(1+u)^{\frac{1}{u}} \qquad (2)$$

Ex 2. Estimate the number e using formula (1) and by taking $n = 100000$.

C Exponential Function

The exponential function e^x may be evaluate using the limit:

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n \qquad (3)$$

Proof:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = \lim_{n\to\infty}\left(\left(1+\frac{x}{n}\right)^{\frac{n}{x}}\right)^x = \left(\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^{\frac{n}{x}}\right)^x =$$

$$= \left(\lim_{u\to 0}(1+u)^{\frac{1}{u}}\right)^x = e^x$$

Ex 3. Estimate $\sqrt{e}$ using formula (2) and by taking $n = 100000$.

D Derivative of e^x

$$(e^x)' = e^x$$
$$\frac{d}{dx}e^x = e^x \qquad (4)$$

Proof:

$$\left(1+\frac{x}{n}\right)^n \to e^x \quad \Rightarrow \quad n\left(1+\frac{x}{n}\right)^{n-1}\left(\frac{1}{n}\right) \to (e^x)'$$

$$\left(\left(1+\frac{x}{n}\right)^n\right)^{\frac{n-1}{n}} \to \left(1+\frac{x}{n}\right)^n \to e^x \quad \Rightarrow \quad \therefore (e^x)' = e^x$$

Ex 4. Differentiate and simplify.

a) $x^2 e^x$

b) $e^{x/2}$

c) e^{-x}

E Derivative of $e^{f(x)}$
Using (4) and the chain rule:

$$(e^{f(x)})' = e^{f(x)} f'(x)$$
$$\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x) \qquad (5)$$

Ex 5. Differentiate.
a) e^{-3x}

b) e^{-1/x^2}

c) $e^{\sqrt{x^2+1}}$

Ex 6. The hyperbolic functions are defined by:
$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\tanh x = \frac{\sinh x}{\cosh x}$.
Prove that:

a) $\cosh^2 x - \sinh^2 x = 1$

b) $(\sinh x)' = \cosh x$

c) $(\cosh x)' = \sinh x$

d) $(\tanh x)' = \frac{1}{\cosh^2 x}$

F Derivative of $b^x, \quad b > 0, b \neq 1$

$$(b^x)' = (\ln b) b^x$$
$$\frac{d}{dx} b^x = (\ln b) b^x \qquad (6)$$

Proof:
$(b^x)' = (e^{x \ln b})' = e^{x \ln b} (\ln b) = (\ln b) b^x$

Ex 7. Differentiate.
a) 3^x

b) $x^2 2^x$

c) $(4^x + x^4)^3$

G Derivative of $b^{f(x)}$
Using (6) and the chain rule:

$$(b^{f(x)})' = (\ln b) b^{f(x)} f'(x)$$
$$\frac{d}{dx} b^{f(x)} = (\ln b) b^{f(x)} f'(x) \qquad (7)$$

Ex 8. Differentiate.
a) 2^{-x^3}

b) $10^{\sqrt{e^x - x^2}}$

Ex 9. Find the equation of the tangent line to the graph of $y = f(x) = x(2^{-x})$ at $(0,0)$.	Ex 10. Find the local extrema for $y = f(x) = x^2 e^{-x^2}$.
Ex 11. Find the points of inflection for $y = f(x) = e^{-x^2}$.	Ex 12. Find the global extrema for $f(x) = x^3 10^{-x}$ over $[-1,2]$.

Reading: Nelson Textbook, Pages 227-232
Homework: Nelson Textbook: Page 232 #2ef, 3cdf, 4abc, 5a, 8, 10a, 13, 16, 17

Reading: Nelson Textbook, Pages 235-239
Homework: Nelson Textbook: Page 240 #1bd, 2abcd, 3, 4, 6, 8, 9