WL02 DIFFRACTION & INTERFERENCE IN 2D

SPH4U

CH 9 (KEY IDEAS)

- analyze and interpret the properties of two-dimensional mechanical waves in a ripple tank and relate them to light
- derive and apply equations involving the speed, wavelength, frequency, and refractive index of waves and apply them to the behaviour light
- analyze two-point-source interference patterns in a ripple tank and in the interference of light (Young's experiment) using diagrams
- derive and apply equations relating the properties of wave interference and wavelength
- outline the historical development of the particle and wave theories of light, including the development of new technologies and discoveries, and summarize the successes and failures of each theory
- apply the wave theory to the property of dispersion and determine the wavelengths of the colours of the visible spectrum

EQUATIONS

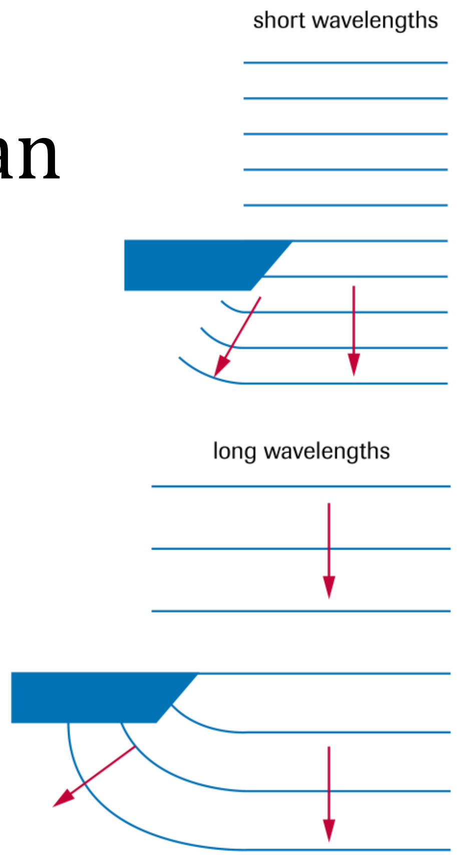
- Two-Point Source Interference

$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

$$\frac{x_n}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

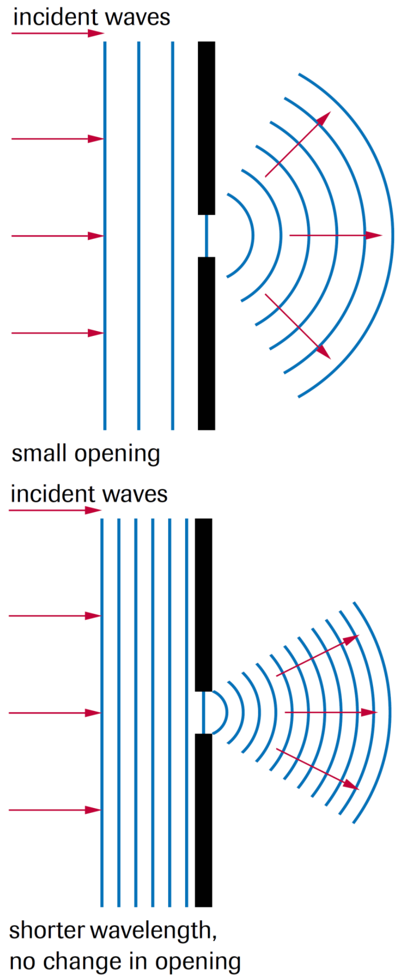
DIFFRACTION OF WATER WAVES

- **Diffraction:** the bending effect on a wave's direction as it passes through an opening or by an obstacle
- The speed of a wave does not influence the diffraction
- Longer wavelengths bend more than shorter wavelengths



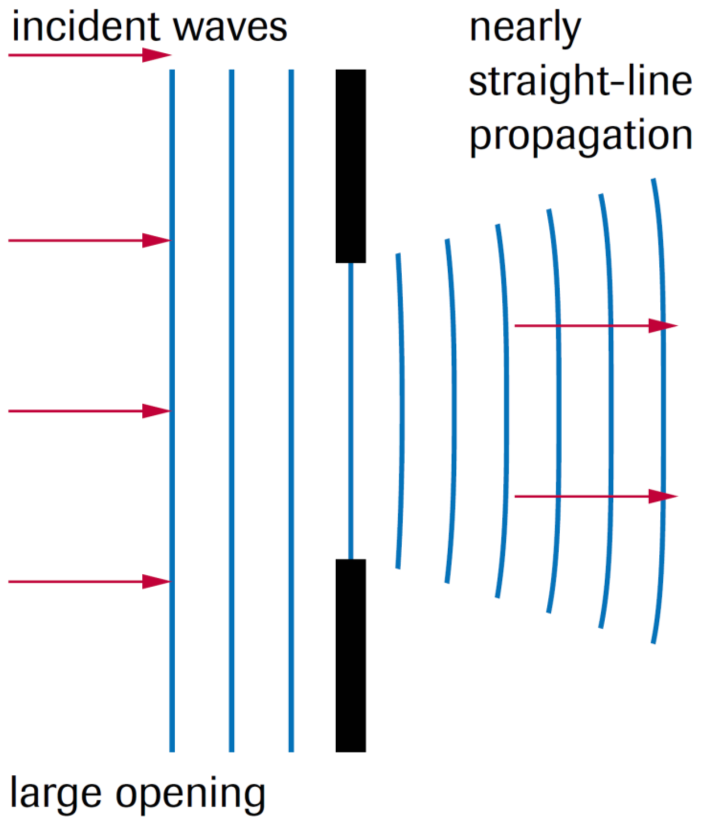
DIFFRACTION OF WATER WAVES: THROUGH AN APERTURE

- Keeping a constant aperture width *w* and changing the wavelength shows how wavelength influences diffraction
- Smallest wavelengths show least diffraction, while longest wavelengths show the greatest diffraction



DIFFRACTION OF WATER WAVES: THROUGH AN OPENING – CONT.

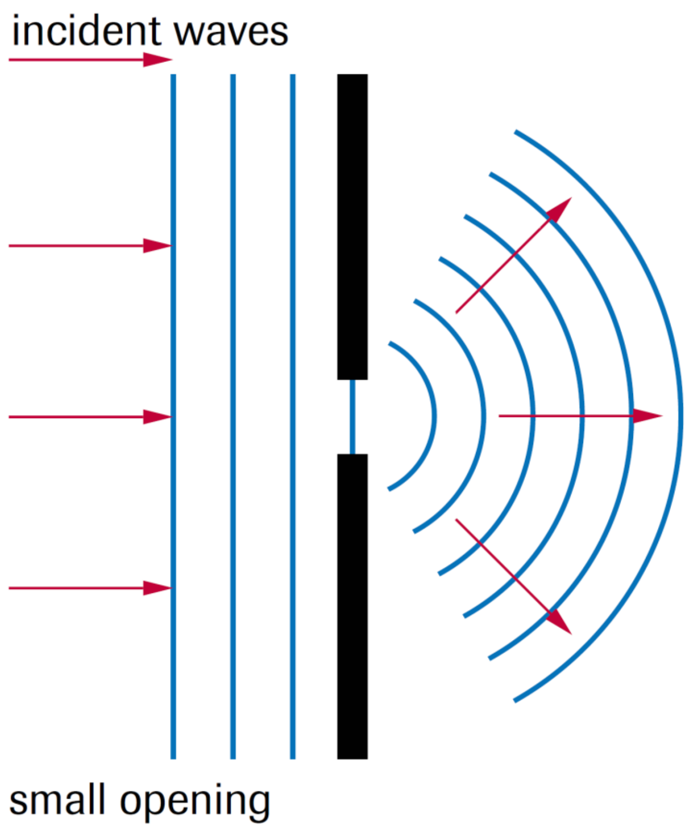
- Keeping a constant wavelength λ lets us explore how a change in aperture width w affects diffraction
- When $w > \lambda$, we notice only diffraction at the edges of the aperture, with no change in the centre of the waves



DIFFRACTION OF WATER WAVES: THROUGH AN APERTURE – CONT.

- When $w \leq \lambda$, the waves are strongly diffracted
- Equivalently, we find the ratio between wavelength and aperture must be greater than or equal to one to produce significant diffraction

$$\frac{\lambda}{w} \geq 1$$

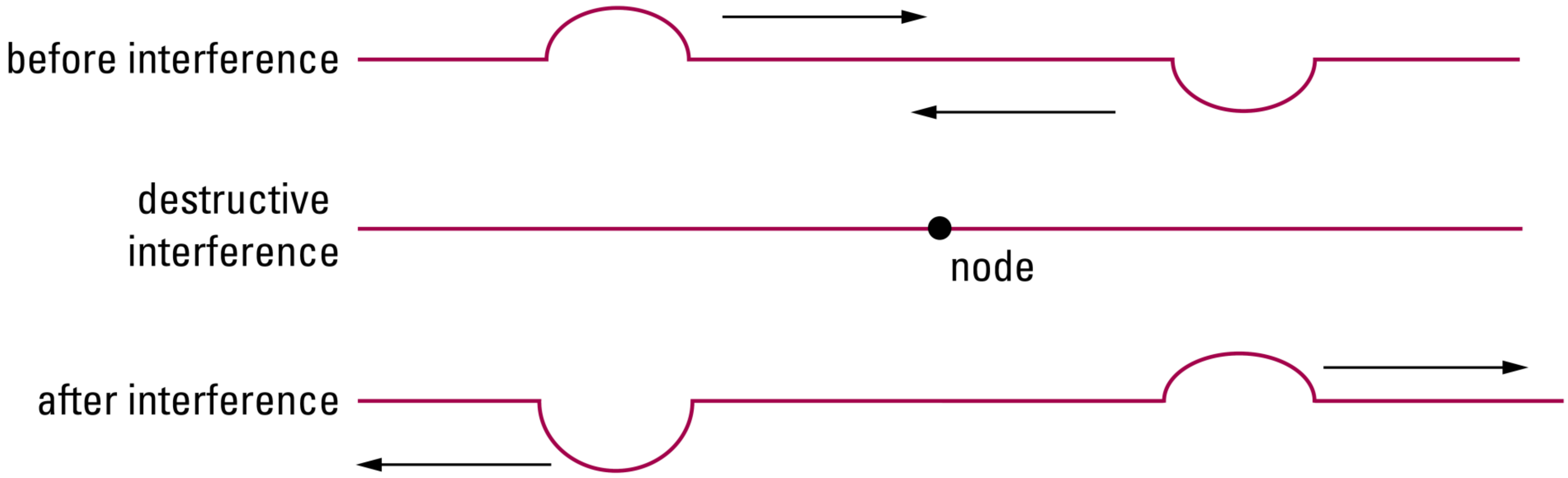


RECALL: INTERFERENCE IN ONE DIMENSION

- **Wave Interference:** occurs when two or more waves act simultaneously on the same particles of a medium
  - **Destructive Interference**
  - **Constructive Interference**
- **Supercrest:** occurs when a crest meets a crest
- **Supertrough:** occurs when a trough meets a trough
- **Principle of Superposition:** At any point the resulting amplitude of two interfering waves is the algebraic sum of the displacements of the individual waves.

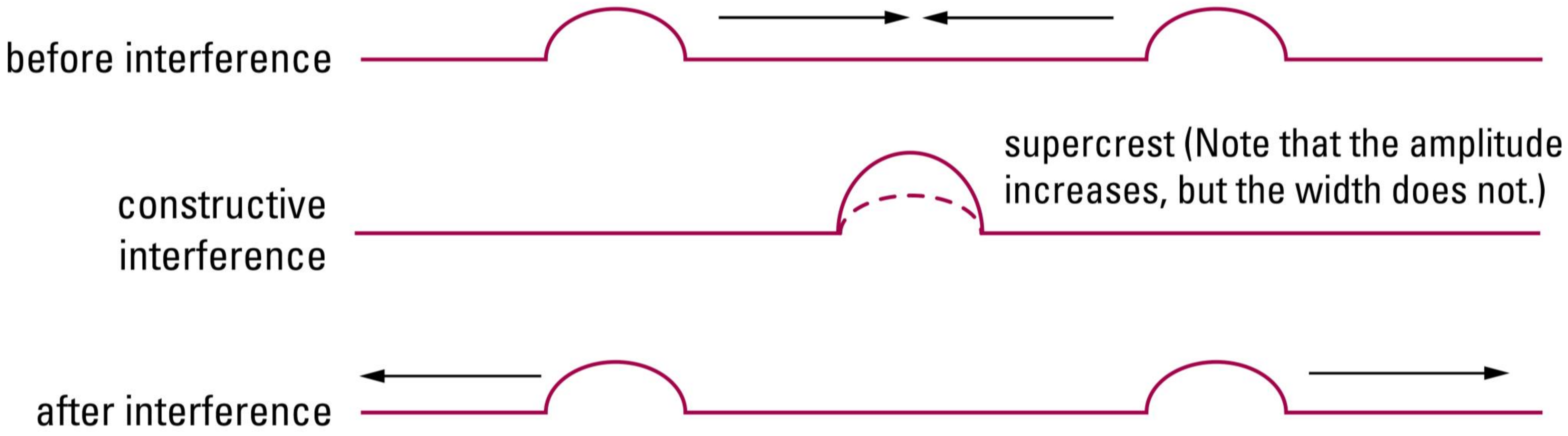
RECALL: 1D WAVE INTERFERENCE – DESTRUCTIVE

- **Destructive Interference:** occurs when waves diminish one another and the amplitude of the medium is less than it would have been for either of the interfering waves acting alone



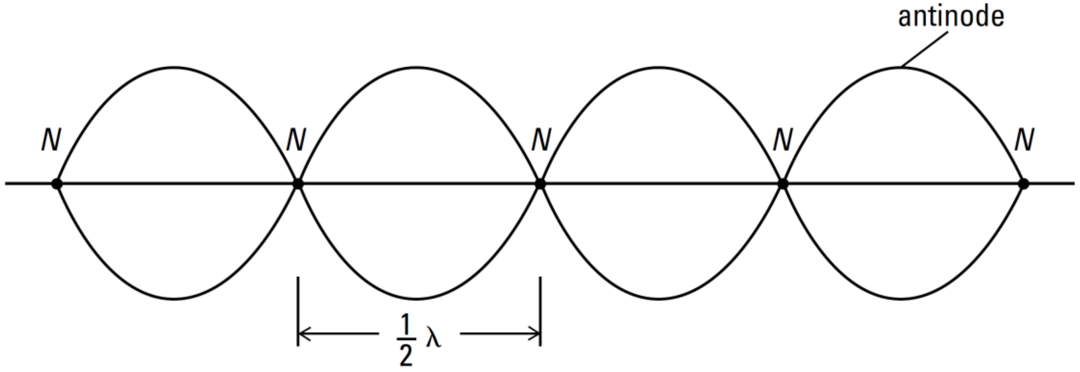
RECALL: WAVE INTERFERENCE – CONSTRUCTIVE

- **Constructive Interference:** occurs when waves build each other up, resulting in the medium having a larger amplitude



FIXED PATTERNS OF INTERFERENCE

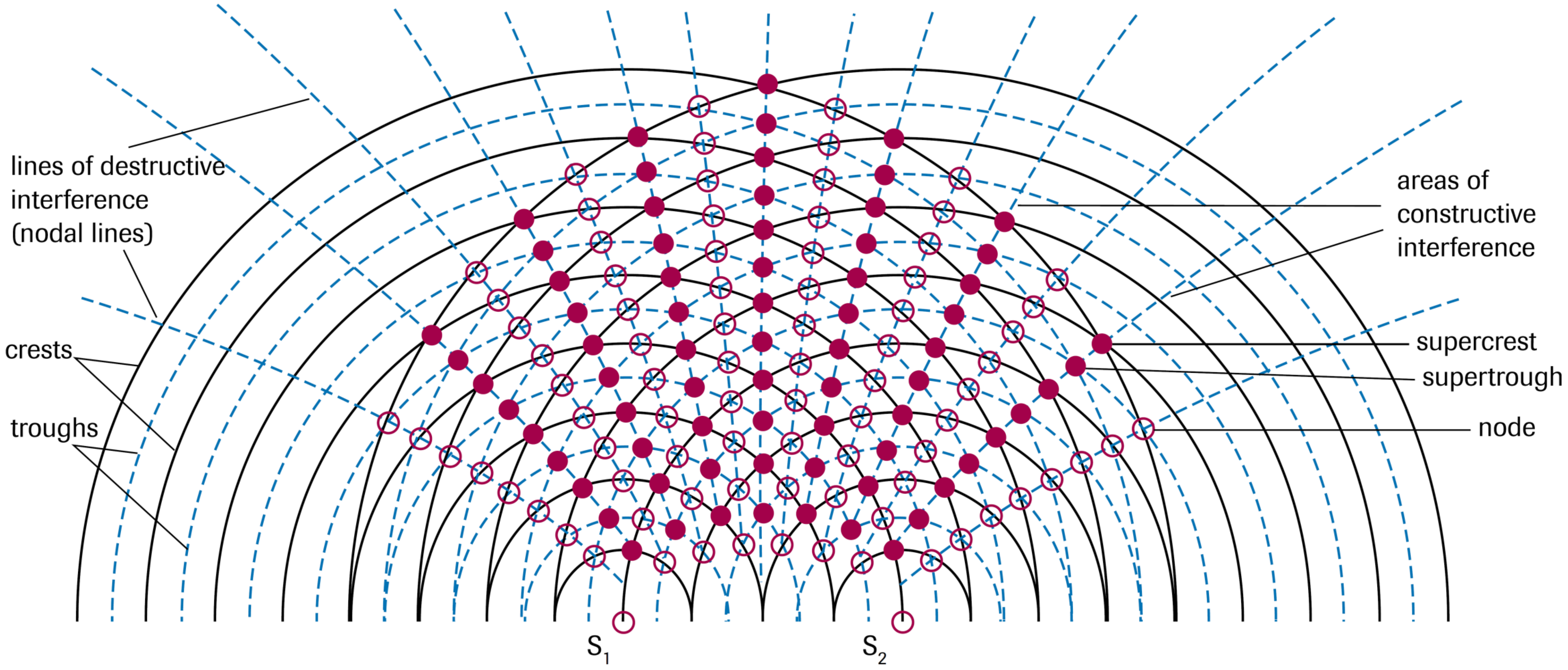
- In order for interference to produce a fixed pattern, waves must
  - Travel at the same speed
  - Have the same frequency (and therefore same wavelength)
  - Have similar amplitudes
- In one dimension, we saw <u>standing waves</u>



INTERFERENCE OF WAVES IN TWO DIMENSIONS

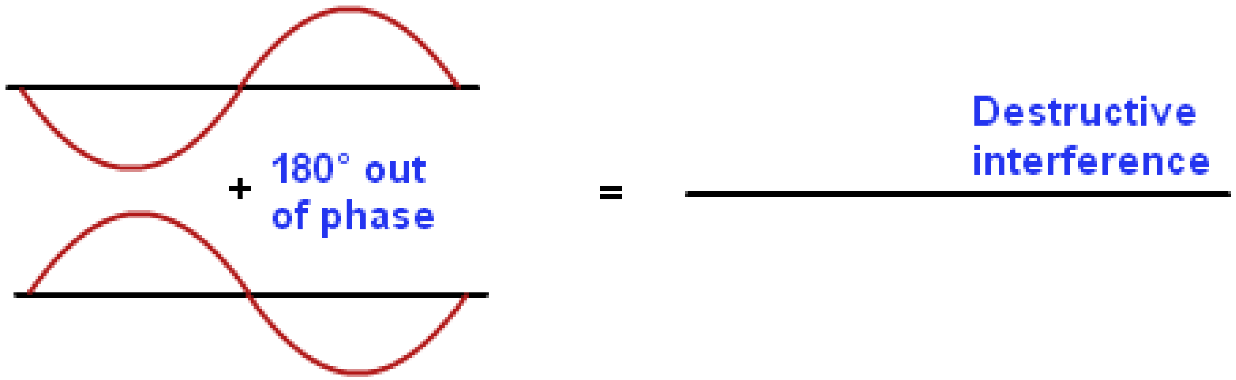
- **Constructive Interference:** occurs when waves build each other up, producing a resultant wave of greater amplitude than the given waves
- **Destructive Interference:** occurs when waves diminish one another, producing a resultant wave of lower amplitude than the given waves
- **Nodal Line:** a line of destructive interference; forms a hyperbolic curve

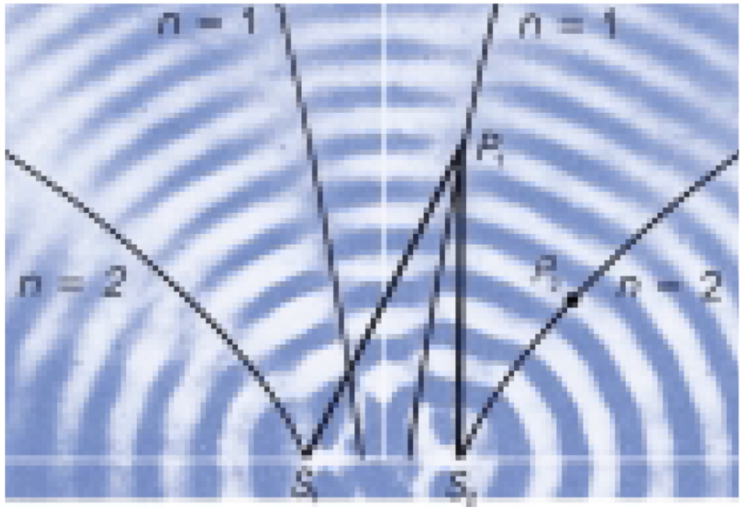
FIXED PATTERNS OF INTERFERENCE: TWO DIMENSIONS



MATHEMATICAL ANALYSIS OF TWO POINT SOURCES

- Consider a point P that lies on one of the nodal lines
- A node occurs when a crest meets a trough
- This occurs when the two waves are out of sync by 180°





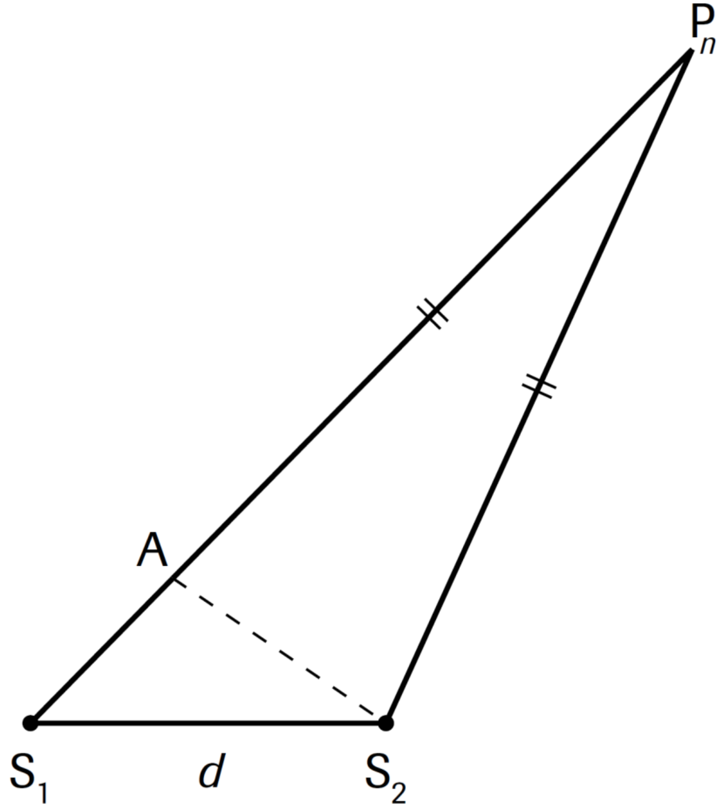
MATHEMATICAL ANALYSIS OF TWO POINT SOURCES

- **Difference in Path Length:** in an interference pattern, the absolute value of the difference between the distance of any point P from one source and the distance of the same point P from the other source:

$$|P_1S_1 - P_1S_2| = \frac{1}{2}\lambda$$

$$|P_2S_1 - P_2S_2| = \frac{3}{2}\lambda$$

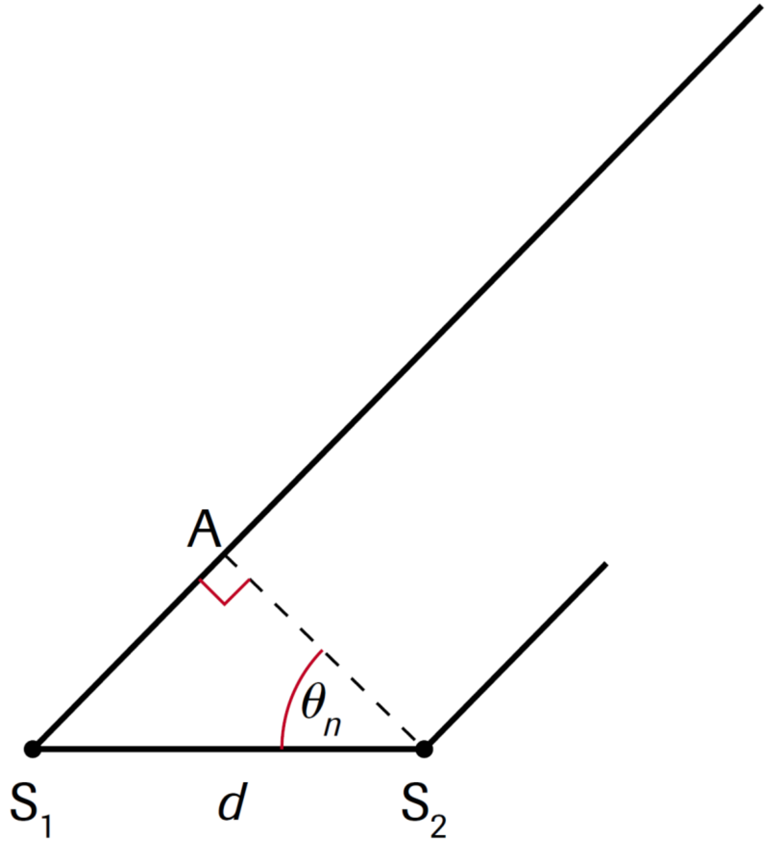
$$|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda \quad \text{(Eq 1)}$$



MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- When the wavelengths are too small or P is too far away from the source, we cannot measure the lengths accurately
- We approximate using angles and the path difference
- For any point P_n, the path length difference is AS_1, so

$$|P_nS_1 - P_nS_2| = AS_1$$



MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

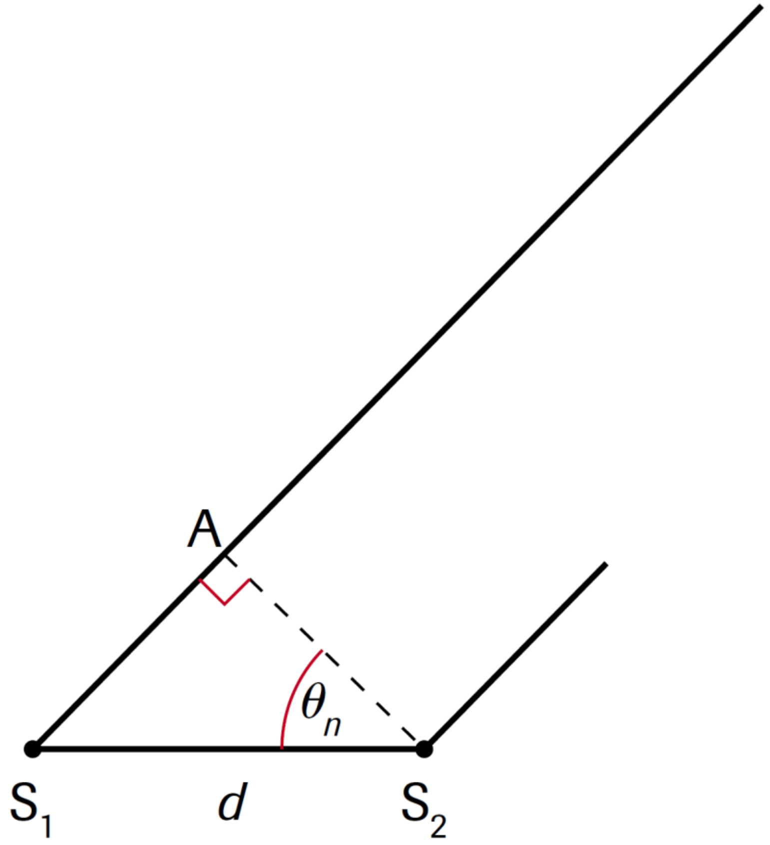
- When P_n is very far compared to source separation d, we can approximate

$$P_n S_1 \parallel P_n S_2$$

- This means $AS_1 = P_n S_2$, so $\triangle S_1 S_2 A$ is a right triangle
- The path difference can now be

$$\sin\theta_n = \frac{AS_1}{d}$$

$$AS_1 = d\sin\theta_n \quad \text{(Eq 2)}$$

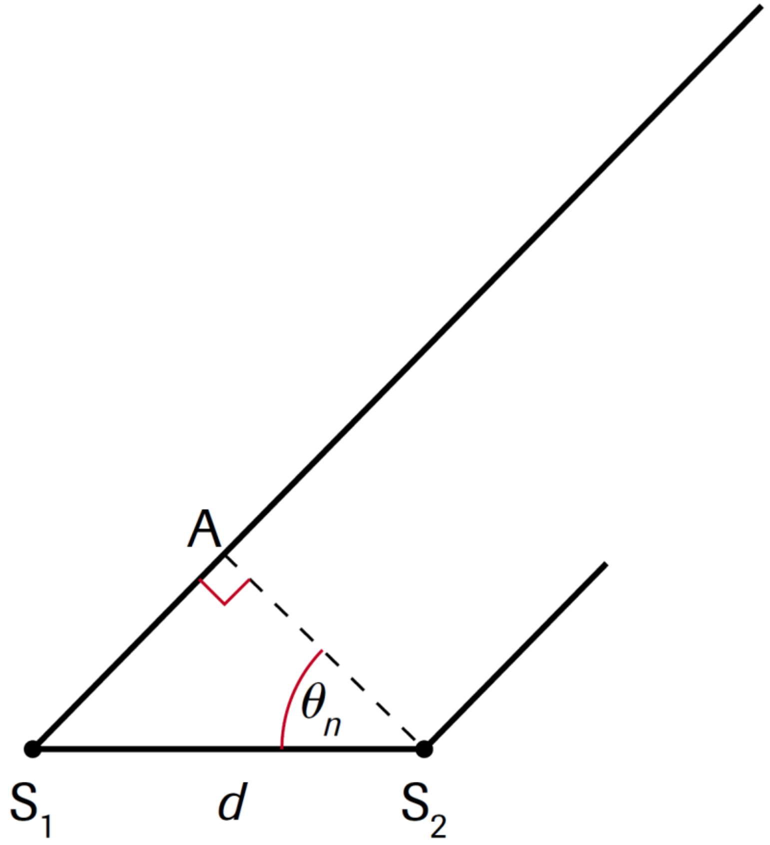


MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- We now know
  1. $|\mathrm{P}_n\mathrm{S}_1 - \mathrm{P}_n\mathrm{S}_2| = \left(n - \frac{1}{2}\right)\lambda$
  2. $\mathrm{AS}_1 = d \sin\theta_n$
- Combining these equations gives

$$\boldsymbol{\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}}$$

  - θ_n – the angle for the n^{th} nodal line
  - λ – wavelength [m]
  - d – distance between sources [m]



MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- NOTE: since the maximum value for $\sin\theta_n$ is 1, the maximum value for $\left(n - \frac{1}{2}\right)\frac{\lambda}{d}$ is also 1
- We find the maximum n-value by counting nodal lines on one side of the right bisector (center anti-nodal line)
- Example: if $n = 4$ and $d = 2.0$ m

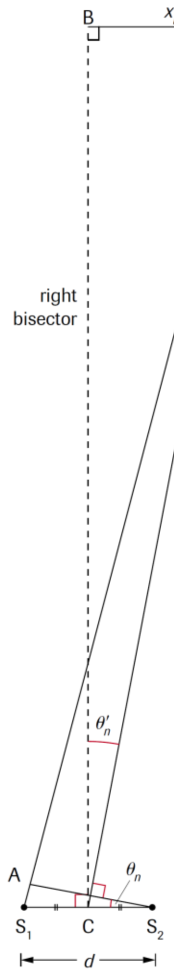
$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d} \approx 1$$

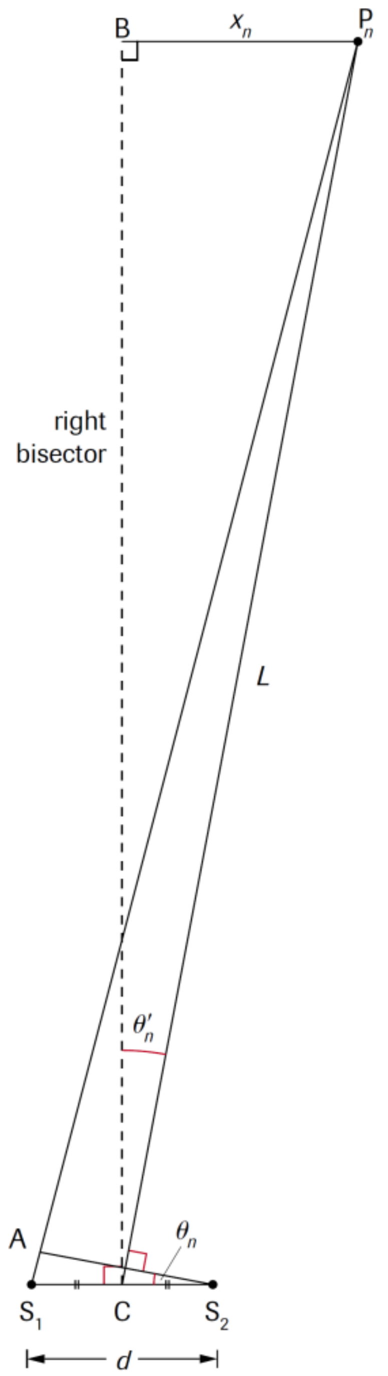
$$\left(4 - \frac{1}{2}\right)\frac{\lambda}{2.0} \approx 1$$

$$\lambda \approx 0.57 \text{ cm}$$

MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- While it is relatively easy to measure angles for water waves, it is impossible to measure the angles for light waves
- We need a technique to find $\sin\theta_n$ without knowing θ_n





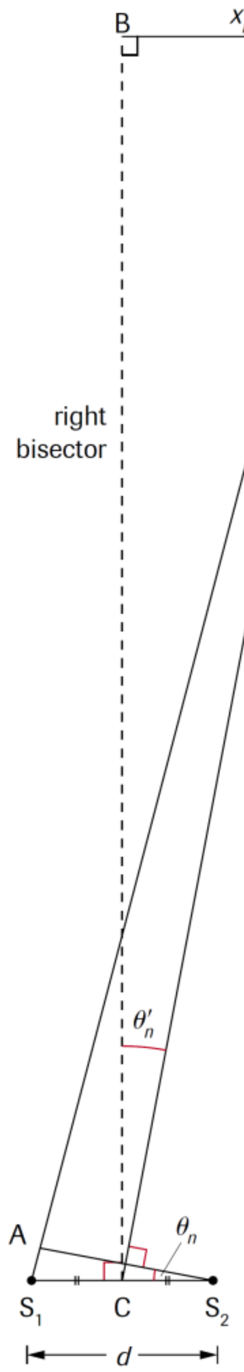
MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- For a point P_n on a nodal line, with the point at the centre between the sources C, we can say

$$P_nC \parallel P_nS_1$$
$$P_nC \perp AS_1$$

- With θ'_n as the angle between P_nC and the right bisector CB, $P_nB = x_n$ and $P_nC = L$,

$$\sin\theta'_n = \frac{x_n}{L}$$



MATHEMATICAL ANALYSIS OF TWO POINT SOURCES – CONT.

- Since $\sin\theta'_n = \sin\theta_n \approx 1$

$$\boldsymbol{\frac{x_n}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}}$$

  - x_n – perpendicular distance from the right bisector to point P_n [m]
  - L – distance from P_n to midpoint C [m]
  - n – number of nodal line, as counted from right bisector
  - λ – wavelength [m]
  - d – source separation [m]

PROBLEM 1

The distance from the right bisector to the second nodal line in a two-point interference pattern is 8.0 cm. The distance from the midpoint between the two sources to point P is 28 cm. What is angle θ_2 for the second nodal line?

PROBLEM 1 – SOLUTIONS

$x_2 = 8.0 \text{ cm}$

$L = 28 \text{ cm}$

$\theta_2 = ?$

$$\sin\theta_2 = \frac{x_2}{L}$$

$$= \frac{8.0 \text{ cm}}{28 \text{ cm}}$$

$$\theta_2 = 16.6°, \text{ or } 17°$$

The angle θ_2 for the second nodal line is 17°.

PROBLEM 2

Two identical point sources 5.0 cm apart, operating in phase at a frequency of 8.0 Hz, generate an interference pattern in a ripple tank. A certain point on the first nodal line is located 10.0 cm from one source and 11.0 cm from the other. What is (a) the wavelength of the waves and (b) the speed of the waves?

PROBLEM 2 – SOLUTIONS

$d = 5.0\text{ cm}$ $\quad PS_2 = 10.0\text{ cm}$ $\quad \lambda = ?$

$f = 8.0\text{ Hz}$ $\quad PS_1 = 11.0\text{ cm}$ $\quad v = ?$

(a)

$$|PS_1 - PS_2| = \left(n - \frac{1}{2}\right)\lambda$$

$$|11.0\text{ cm} - 10.0\text{ cm}| = \left(1 - \frac{1}{2}\right)\lambda$$

$$\lambda = 2.0\text{ cm}$$

The wavelength of the waves is 2.0 cm.

(b)

$$v = f\lambda$$

$$= (8.0\text{ Hz})(2.0\text{ cm})$$

$$v = 16\text{ cm/s}$$

The speed of the waves is 16 cm/s.

Summary:
Diffraction of Water Waves

- Waves diffract when they pass by an obstacle or through a small opening.
- Waves of longer wavelength experience more diffraction than waves with a smaller wavelength.
- For a given opening or aperture, the amount of diffraction depends on the ratio $\frac{\lambda}{w}$. For observable diffraction, $\frac{\lambda}{w} \geq 1$.

SUMMARY: INTERFERENCE OF WAVES IN TWO DIMENSIONS

- A pair of identical point sources operating in phase produces a symmetrical pattern of constructive interference areas and nodal lines. The nodal lines are hyperbolas radiating from between the two sources.
- Increasing the frequency (lowering the wavelength) of the sources increases the number of nodal lines.
- Increasing the separation of the sources increases the number of nodal lines.
- Changing the relative phase of the sources changes the position of the nodal lines but not their number.
- The relationship $\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$, or $\frac{x_n}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$, can be used to solve for an unknown in a two-point-source interference pattern.

PRACTICE

Readings

- Section 9.2 (pg 453)
- Section 9.3 (pg 455)

Questions

- pg 454 #1-4
- pg 460 #1-5,7,9